MATH 117 – FINAL EXAM

Acad.Year : *2016-2017*	Last Name :	
Semester : *Spring*	Name :	
Coord. : *B.Okutmustur*	Student id :	
Date : *01.06.2017*	Signature :	
Time : *13:30*	5 QUESTIONS ON 4 PAGES	
Duration : *120 min*	TOTAL 100 POINTS	

1. (20)	2. (24)	3. (24)	4. (20)	5. (12)	

Please draw a $\boxed{\text{box}}$ around your answers. **No** calculators, cell-phones, notes, etc. allowed.

1. (20 pts.) Evaluate the following

(a) $\dfrac{d}{dt}\left[\underbrace{\displaystyle\int_{\sqrt{x}}^{x} \sqrt{t}\sin(t^4)\,dt}_{\text{"depends on } x\text{"}}\right] = \boxed{0}$ since the integral depends on "x" and derivative is with respect to "t"

(b) $\displaystyle\lim_{x\to 0} \frac{\int_0^{\sin 3x} \cos(7t)\,dt}{\sin x}$ when $x \to 0$, we get $\left(\frac{0}{0}\right)$ form. So we apply L'Hospital Rule and FTC

$$\overset{L'H}{\underset{FTC}{=}} \lim_{x\to 0} \frac{3 \cdot \cos 3x \cdot \cos(7\sin 3x)}{\cos x} = \frac{3\cos 0 \cdot \cos(\sin 0)}{\cos 0} = \boxed{3}$$

(c) Find the local maximum/minimum point(s) of the function $f(x) = \displaystyle\int_0^{2x^2-x^4} \sin\left(\frac{1}{1+t^2}\right) dt$

Local extrema may occur at the critical points of f, i.e. $f'(x)=0$

By FTC : $f'(x) = (4x-4x^3)\sin\left(\dfrac{1}{1+\underbrace{(2x^2-x^4)^2}_{>0}}\right) = 0$ $\Rightarrow 1+(2x^2-x^4)^2 \neq 0$ $\Rightarrow \sin\left(\frac{1}{1+(2x^2-x^4)^2}\right) \neq 0$

$\Rightarrow 4x-4x^3 = 0 \Rightarrow 4x(1-x^2) = 0$

$\Rightarrow x=0,\ x=-1$ and $x=1$ are the critical points

	-1		0		1	
$+$		$-$		$+$		$-$
	loc. max		loc min		loc max	

f has loc max at $\boxed{x=-1}$ and $\boxed{x=1}$

f has loc min at $\boxed{x=0}$

2. (24 pts.) Evaluate the following integrals

(a) $\int \sin(x)\,3^{\cos(x)}\,dx$ $= -\int 3^u\,du = -\dfrac{3^u}{\ln 3} + C$

Let $u = \cos x$

$du = -\sin x\,dx$

$= \boxed{-\dfrac{3^{\cos x}}{\ln 3} + C}$

C: constant

(b) $\int_{-\frac{1}{5}}^{0} \frac{x}{\sqrt[5]{5x+1}}\,dx$

Let $u^5 = 5x+1$ and $x = \dfrac{u^5-1}{5}$

$\Rightarrow 5u^4\,du = 5\,dx$

$\Rightarrow u^4\,du = dx$

For $x = -\frac{1}{5} \Rightarrow u = 0$

For $x = 0 \Rightarrow u = 1$

$$= \int_0^1 \frac{u^5-1}{5}\cdot\frac{\cancel{u^4}\,du}{\cancel{u}} = \frac{1}{5}\int_0^1 u^3(u^5-1)\,du$$

$$= \frac{1}{5}\int_0^1 (u^8-u^3)\,du = \frac{1}{5}\left(\frac{u^9}{9}-\frac{u^4}{4}\right)\Bigg|_0^1$$

$$= \frac{1}{5}\left(\frac{1}{9}-\frac{1}{4}\right) = \frac{1}{5}\left(\frac{-5}{36}\right) = \boxed{-\frac{1}{36}}$$

(c) $\int \tan^5 x \sec^3 x\,dx$ $= \int \tan^4 x \sec^2 x \underbrace{\tan x \sec x\,dx}_{du}$

Let

$u = \sec x$

$du = \sec x \tan x\,dx$

and

$\sec^2 x = 1 + \tan^2 x$

$\Rightarrow u^2 = 1 + \tan^2 x$

$\Rightarrow \tan^2 x = u^2 - 1$

$$= \int (u^2-1)^2 u^2\,du$$

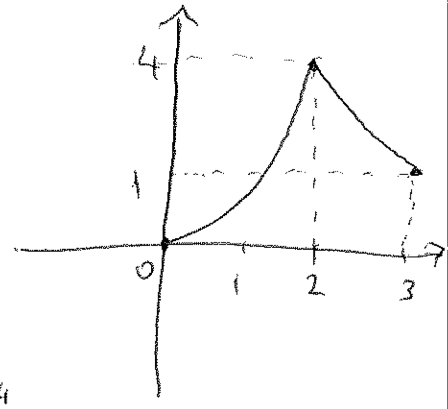
$$= \int (u^4 - 2u^2 + 1)\,u^2\,du$$

$$= \int (u^6 - 2u^4 + u^2)\,du$$

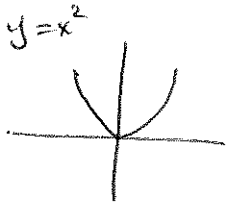
$$= \frac{u^7}{7} - 2\frac{u^5}{5} + \frac{u^3}{3} + C$$

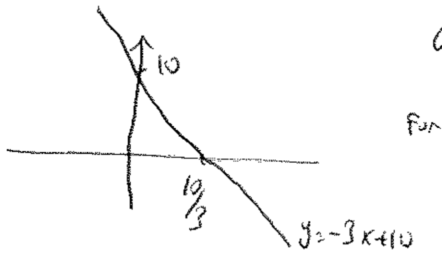
$$= \boxed{\frac{(\sec x)^7}{7} - 2\frac{(\sec x)^5}{5} + \frac{(\sec x)^3}{3} + C}$$

3. (24 pts.) Given $f(x) = \begin{cases} x^2 & \text{if } 0 \leqslant x < 2 \\ -3x + 10 & \text{if } 2 \leqslant x \leqslant 3 \end{cases}$



(a) Sketch the graph of f on the interval $[0, 3]$





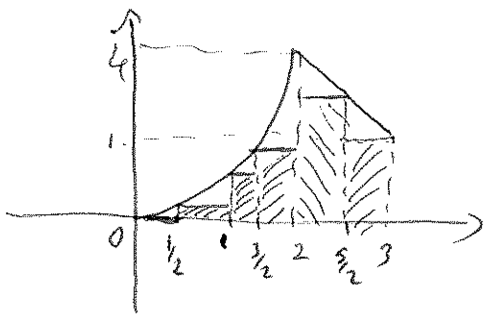
Combine both graphs

For $x=2 \Rightarrow y=-3x+10=4$

$x=3 \Rightarrow y=-3x+10=1$

(b) Compute the Lower Riemann sum for the function $f(x)$ on $[0, 3]$ into **six** subintervals of equal length.

$$\Delta x = \frac{3-0}{6} = \frac{1}{2}$$



$$L(f,P) = \Delta x \left(f(0) + f\left(\tfrac{1}{2}\right) + f(1) + f\left(\tfrac{3}{2}\right) \right) + \Delta x \left(f\left(\tfrac{5}{2}\right) + f(3) \right)$$

$$= \frac{1}{2}\left(0 + \frac{1}{4} + 1 + \frac{9}{4}\right) + \frac{1}{2}\left(\frac{5}{2} + 1\right)$$

$$= \boxed{\frac{7}{2}}$$

(c) Calculate $\int_0^3 f(x)dx$ and compare with the result in (b)

$$= \int_0^2 x^2 \, dx + \int_2^3 (-3x+10)\, dx = \frac{x^3}{3}\Big|_0^2 + \left(-\frac{3x^2}{2} + 10x\right)\Big|_2^3$$

$$= \frac{8}{3} + \left(-\frac{27}{2} + 30\right) - \left(-6 + 20\right) = \boxed{\frac{31}{6}} > \boxed{\frac{7}{2}} = L(f,P)$$

(d) Evaluate the mean (average) value of f on $[0, 3]$

$$\bar{f} = \frac{1}{3-0} \int_0^3 f(x)\, dx = \frac{1}{3} \cdot \frac{31}{6} = \boxed{\frac{31}{18}}$$

4. (20 pts.) Express the following limit as a definite integral and evaluate it

$$\lim_{n\to\infty}\left[\frac{e}{n}\left(e^{\frac{1}{n}}+e^{\frac{2}{n}}+....+e^{\frac{n}{n}}\right)\right] = \lim_{n\to\infty} e\, S_n \quad \text{where}$$

$$S_n = \sum_{i=1}^{n} \frac{1}{n}\left(e^{\frac{i}{n}}\right)$$

$\Delta x = \frac{1}{n}$, $x_1 = \frac{1}{n}$, $x_2 = \frac{2}{n}$, ..., $x_n = \frac{n}{n} = 1$

whenever $n\to\infty$, $x_1 \to 0 = a$, $x_n \to 1 = b$ $\Rightarrow$ $\boxed{a=0,\ b=1}$

and $x_i = \frac{i}{n}$ $\Rightarrow$ $f(x_i) = e^{x_i}$ $\Rightarrow$ $f(x) = e^x$

$$\Rightarrow \lim_{n\to\infty} e\, S_n = e \lim_{n\to\infty} S_n = e\int_0^1 e^x\,dx$$

$$= e \cdot e^x \Big|_0^1 = \boxed{e(e-1)}$$

5. (12 pts.) Is the function $g(x) = \begin{cases} 2x^2-1, & \text{if } x \geqslant 0 \\ 2x^2+1, & \text{if } x<0 \end{cases}$ differentiable at $x=0$? Explain.

We first check continuity of g at $x=0$.

$$\left.\begin{aligned} \lim_{x\to 0^+} g(x) &= \lim_{x\to 0^+} 2x^2-1 = -1 \\ &\qquad\qquad\qquad\qquad \neq \\ \lim_{x\to 0^-} g(x) &= \lim_{x\to 0^-} 2x^2+1 = 1 \end{aligned}\right\} \quad g(x) \text{ is } \underline{\text{NOT}} \text{ continuous at } x=0$$

$\Rightarrow$ g is $\underline{\text{NOT diff'ble}}$ at $x=0$